Problem Set 8: Complex Numbers

<u>Goal</u>: Become familiar with math operations using complex numbers; see how complex numbers can be used to show the frequency response of an RC circuit.

Note: This PSet will be much easier if you have already watched the lectures on complex numbers.

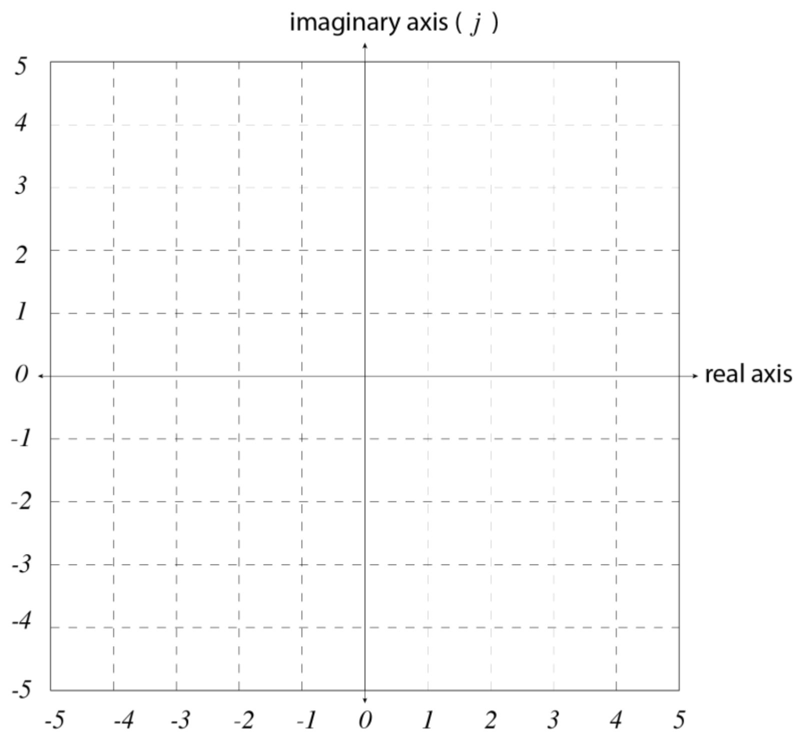
Deliverable: This worksheet and two plots.

Part I: Basic Operations with complex numbers

For the following, take $z_1 = 1 + j$ and $z_2 = -3 + 4j$.

1. Convert z_1 and z_2 to polar and exponential notation (find r, θ).

2. Plot z_1 and z_2 on the complex plane below.



3. Compute z_1+ z_2. Show z_1+ z_2. graphically on a plot in the complex plane from 2.

4. Compute z_1- z_2. Show z_1- z_2. graphically on a plot in the complex plan from 2.

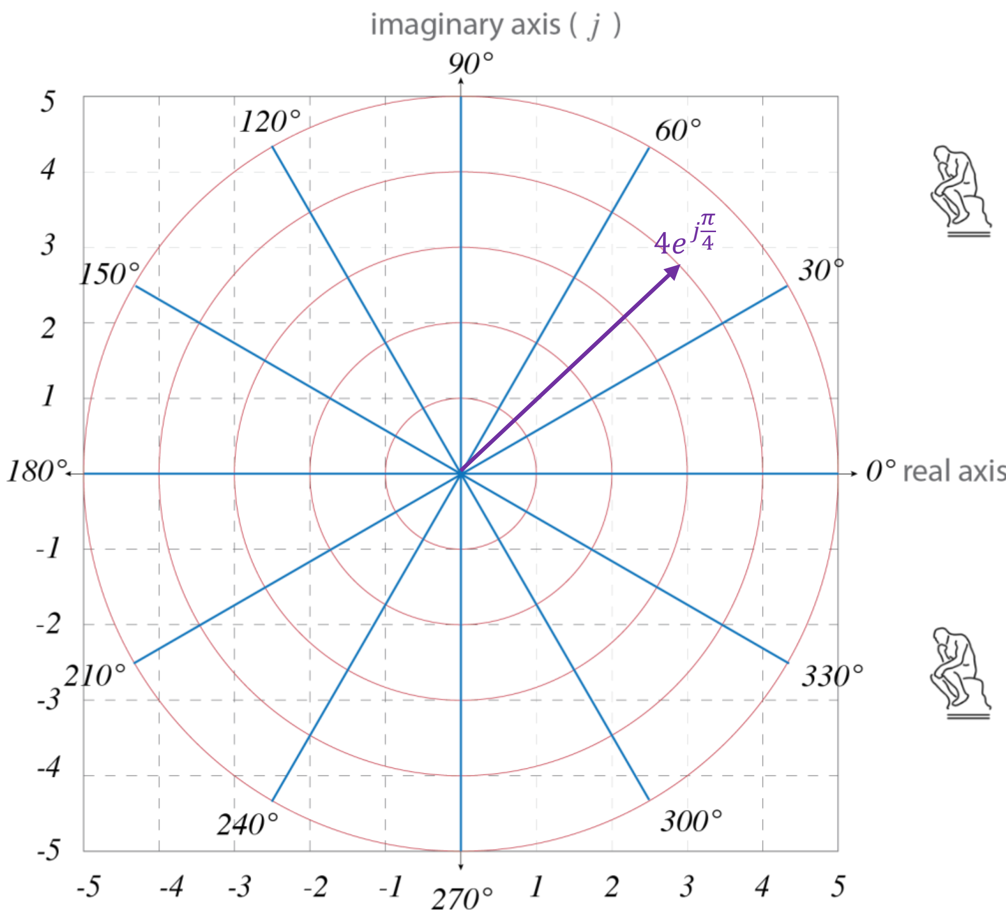
5. Compute $z_1 z_2$. Repeat the computation using a different notation.

6. Compute ${z_1}/{z_2}$ using complex notation. Compute ${z_2}/{z_1}$ and compare.

7. Compute z_1^4

Part II: Plotting complex numbers

Complex numbers using **polar notation** are super useful for illustrating how a circuit responds to time-varying signals.



What does $3 \cdot e^{j\pi}$ look like?

A Wavegen input signal is a sine wave, $V_{peak-peak} = 4Volts.$ In r, θ notation, what is its r ?

The **polar coordinates** (above grid of red & blue) make use of a special property of the ***exponential function*** when it operates on $j(=\sqrt{-1})$**.** You may have seen this function notated (equivalently) as:

$$e^{j\theta}, \exp{(j\theta)}, \text{ or } e^{i\theta}$$

where θ represents an angle in radians (Recall that π radians = 180°).

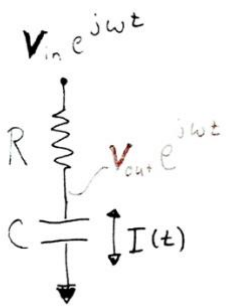
The amazing property of $e^{j\theta}$ is known as Euler's formula (section 6.3 in your book):

$$e^{j\theta} = \cos(\theta) + jsin(\theta)$$

If $\boldsymbol{\theta}$ varies with a frequency, $\boldsymbol{\omega}, \boldsymbol{\theta} = \boldsymbol{\omega} \cdot \boldsymbol{t}$, what would $\boldsymbol{e^{j\omega t}}$ look like in time?

Click this link to see. There is more info on page 4 for those who are interested.

Recall from Figure 6.3 that if we represent our cosine voltage input to a **low-pass filter** with polar notation,



$V_{in}(t) = \boldsymbol{V}_{in} \cdot e^{j\omega t}$

And $\boldsymbol{V}_{in}$ represents a complex number.

And remember that because the R and C are in series, the time varying current passing through both will be the same, we get,

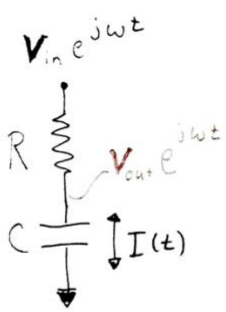
$$\frac{V_{in}(t) - V_{out}(t)}{R} = C\frac{dV_{out}(t)}{dt}$$

and, rearranged a bit,

$$\boldsymbol{V}_{in} \cdot e^{j\omega t} - \boldsymbol{V}_{out} \cdot e^{j\omega t} = RCj\omega \boldsymbol{V}_{out} \cdot e^{j\omega t}.$$

Or

solving for $\frac{\boldsymbol{V}_{out}}{\boldsymbol{V}_{in}}$,



$$\frac{\boldsymbol{V}_{out}}{\boldsymbol{V}_{in}} = \frac{1}{1 + j\omega RC}$$

Let's let RC=1 second and $z_3 = \frac{1}{1+j\omega}$

And $z_4 = \frac{j\omega}{1+j\omega}$

Convert z_3 and z_4 to r, θ notation.

Plot the magnitude of *r* of z_3 and z_4 as a function of ω on a log-log scale. Let ω* vary from 10^{-3} to 10^{3}.

Plot θ in degrees for z_3 and z_4 as a function of ω on a semilog scale. Let ω vary from 10^{-3} to 10^{3}.

*In Matlab, you can use the command:
y= logspace(-3,3)
to generate a logarithmically-spaced vector, y, that spans 10^{-3} to 10^{3}.

Knowing that z_3 and z_4 represent the $\frac{V_{out}}{V_{in}}$ of low- and high-pass filters, what do you expect the graphs to look like?

Watch $e^{j\omega t}$ *vary as* $\boldsymbol{\theta}$ *varies with a frequency,* $\boldsymbol{\omega}$: $\boldsymbol{\theta = \omega \cdot t}$

Click this [link](#) to see.

These images illustrate for r=1,

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$

at different points in time $(\theta = \omega \cdot t)$